Growth, export externalities, and accumulation of foreign reserves *

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Abstract

Previous papers have argued that countries accumulate foreign reserves in order to deteriorate terms of trade to increase welfare. On the other hand, the optimal tariff theory argues that tariffs can increase the welfare of a country by improving its terms of trade. This paper provides a plausible explanation for the different foreign reserves policies regarding terms of trade. I build an endogenous growth model of a small open economy with technological spillovers generated from exports. Internalizing the growth effects from these externalities, the government decides whether to accumulate foreign reserves or to borrow from abroad. This paper finds that when the export externalities are large enough, it is optimal to hold positive foreign reserves to achieve faster growth through terms of trade deterioration. However, when the export externalities are small, the combined effects from the consumption smoothing motivation and better terms of trade outweigh the growth effects of exports, so the government holds negative foreign reserves.

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1 Introduction

The accumulation of foreign reserves is a prevalent phenomenon in developing countries. Figure(1) shows the quartiles of foreign reserves to GDP ratio. About half of the sample countries have foreign reserves that exceed 15% of GDP in the early 2000s. Considering that the average size of foreign reserves in 1980 was only 5% of GDP, this is a remarkable growth. A sizable literature offers different arguments for the accumulation of foreign reserves in developing countries, one of them being manipulation of terms of trade (real exchange rate).

Accumulation of foreign reserves can be used to make the real exchange rate depreciate, which is suggested by Benigno and Fornaro (2012). In their paper, economic growth is stimulated by importing intermediate inputs from abroad since they can take advantage of technological spillovers through imports. In this case, the government has an incentive to accumulate foreign reserves to induce real exchange rate depreciation. This leads to higher imports of intermediate goods and thus faster economic growth can be achieved. In this paper, I adopt their framework and incorporate export externalities instead of import externalities. Hence, the government considers the faster growth through exports and thus has an incentive to hold foreign reserves to generate terms of trade deterioration. As a result, the economy exports more and experiences faster growth. This model can provide the justification of the behavior of the Asian Tigers (Hong Kong, South Korea, Taiwan, and Singapore) and China. They are well known for experiencing rapid economic growth using export expansion strategies and also have been holding significant amount of foreign reserves.

However, there are counter-arguments regarding terms of trade policies: optimal tariff theory and consumption smoothing motivation. The optimal tariff theory has been developed in different ways (Kaldor (1940), Johnson (1953-54), Hamilton and Whalley (1983), Kennan and Riezman (1988), Helpman and Krugman (1989), and Syropoulos (2002)) since Bickerdike (1906). The main argument starts from governments facing monopoly/monopsony power. They have an incentive to impose tariffs which induces better terms of trade and increases their welfare at the cost of other countries' welfares. In these models, the opti-

mal tariff level is strategically chosen to achieve the highest welfare of the home country. Also, consumption smoothing motivation in the growth model of a small open economy implies improvement of terms of trade. Expecting higher income, agents would borrow from abroad to smooth their consumption, which is equivalent to more consumption of the imported goods. This results in better terms of trade, so the economy would benefit from consumption smoothing.

Considering these two forces related to the terms of trade, I build a model that can analyze which effect is dominant. I put the optimal tariff policy and consumption smoothing motivation together as a force of improvement of the terms of trade. Since price policy instruments such as tariffs or subsidies may be forbidden by the WTO or conflict with other trade agreements, I limit the policy tool for the management of the terms of trade to foreign reserves, which is not against these trade rules. In other words, when an economy accumulates foreign reserves, it means that the government buys foreign bonds, which are external savings. In this situation, the terms of trade are worse. On the other hand, the government can sell foreign bonds, which is external borrowing, and improve the terms of trade. This is equivalent to the effects of the combination of the optimal tariff policy and the consumption smoothing motivation. Therefore, this model suggests answers to the question why some countries accumulate foreign reserves while some follow the optimal tariff policy.

To address this question, this paper proposes an endogenous growth model of a small open economy with technology spillovers through exports. There are two goods in this economy: domestic and foreign goods. The domestic goods are tradable, and thus can be exported or consumed domestically. The main feature of this model is the export externalities. To export domestic goods, the firms must compete in the foreign market, which allows the firms to access a wide range of technology enhancements through their activities in the foreign market. (De Melo (1985)) These technological innovations are accumulated as a form of knowledge. Higher knowledge implies higher productivity when producing domestic goods. Due to the export externalities the economy can grow faster as it exports more.

This paper shows how governments in exporting countries use foreign reserves to increase their welfares and their optimal decisions depend on the magnitude of export externalities. If the export externalities are large enough, then the government accumulates foreign reserves to achieve faster economic growth. In the model, private agents fail to internalize the export externalities. Therefore, they export less than the socially optimal amount and the economy grows more slowly. On the other hand, the government takes externalities into account, and increases foreign reserves to allow the private sector to export more. The key assumption here is that the foreign reserves are denominated in foreign goods. Thus, the accumulation decision for the foreign reserves can change the terms of trade.[1] Increasing foreign reserves makes the domestic goods relatively cheaper so the domestic country can export more compared to the case where the government does not intervene. As a result, the economy experiences faster growth with technological spillovers through higher exports.

I compare the dynamics of several important variables in two different equilibria. In the competitive equilibrium (CE), consumers are not allowed to save and there is no government, so the foreign reserves are zero over time. However, internalizing the growth effects from export externalities, the government increases foreign reserves, and thus the terms of trade are higher than the one in the CE. This induces more exports, so the optimal consumption is less than the one in the CE for the first few periods. However, because the economic growth is faster, the consumption catches up to the one in the CE, and eventually goes beyond. As a result, the welfare under the government intervention is higher than the one in the CE. This illustrates the intertemporal trade-off from the foreign reserves. The increase in the foreign reserves would lead to less consumption today but more consumption in the future by speeding up growth. Once the economy reaches the steady state, it no longer needs technological innovation through exports, so the government simply maintains enough foreign reserves to stay in the steady state. This is the situation where accumulation of

[1] The terms of trade is defined as the price for imports over the price for exports. In this model, the terms of trade are equal to the real exchange rate since imported goods are foreign goods and exported goods are domestic goods.

foreign reserves improves welfare. With the Korean data, welfare increases by 0.22 percent.

Nevertheless, it is not always optimal to choose to hold positive foreign reserves. To examine this, I vary the magnitude of export externalities and find that the government would borrow when externalities are positive but small. This is because the optimal foreign reserves are determined by comparing the growth effects through exports and the benefits from smoothing consumption and tariff policy. If export externalities are large enough, the former effects outweigh the latter one, so the government sacrifices today's consumption to achieve faster growth by accumulating the foreign reserves. Consequently, the terms of trade deteriorate. But when externalities are small, the latter force is stronger than the former one, so the government borrows to consume more today, which induces better terms of trade. This impedes the growth of the economy, but the economy gains welfare from the benefits of consumption smoothing.

In the extended discussion I allow the consumers to access the foreign bonds market, just as the government could before. This means that the only difference between the government and the private agents is the government's ability of internalizing export externalities. In this competitive equilibrium, the consumers expect higher output in the future, so they would borrow from the foreign country rather than save. This would actually impede the growth of the economy by reducing exports due to lower terms of trade. Therefore, the welfare gains from accumulating foreign reserves could be even larger here if the externalities are large.

Literature review This paper is related to several branches of literature. First, there are papers about growth associated with knowledge spillovers though international trade. The theoretical study of cross-country knowledge spillovers was established by Grossman and Helpman (1991). They build an endogenous growth model of a small open economy and incorporate knowledge flow from abroad through international trade. Since knowledge is non-rivalrous and non-excludable, it is hard to extract compensation from all agents who use it. This is how knowledge brings spillover benefits while trading. The authors aruge

that trade can affect growth in the long run though this mechanism. There is also a sizable literature studying knowledge flow across countries and they show knowledge transmission can be done through either importing foreign products or exporting to foreign markets. I focus on the latter channel in this paper as this channel is supported by other empirical papers.

The current literature provides empirical support for the export promotion policies achieving economic growth. Balassa (1978) and Balassa (1985) show that GDP growth is significantly affected by exports with eleven industrial countries. Tyler (1981) extends this argument to 55 middle income developing countries. He incorporates exports as an input of production function because export externalities may help more efficient production. Feder (1983) first develops a two-sector model with export and non-export sectors and estimate externalities based on the different marginal factor productivities between two sectors. This model indicates that the export sector has higher productivity that leads to growth of the economy by reallocating resources from the non-export sector to the export sector. Bilginsoy and Khan (1994) improve Feder's specification by relaxing some assumptions and measuring the export externalities.[2] All these papers empirically show the positive correlation between economic growth and exports. The externalities are generally mentioned with efficiency from competitive management, the introduction of advanced techniques, training of higher quality labor, or economies of scale. In this paper, I develop a theory of knowledge externalities through exports, which is consistent with key results documented in the empirical literature.

There have been huge studies on foreign reserves. One explanation is that accumulation of foreign reserves is a result of the precautionary motive. Developing economies are concerned about the situation of the limited access to international financial markets. Therefore, they have the purpose of self-insurance. (Ranciere and Jeanne (2006), Aizenman and Lee (2007), Caballero and Panageas (2007), Alfro and Kanczuk (2009), Bianchi, Hatchondo, and Martinez (2016)). Meanwhile there is another factor that accounts for a significant amount

[2]Bilginsoy and Khan (1994) allow intersectoral externality and constant term in their regression and chose non-export sector growth as the dependent variable to avoid spurious correlation problem.

of foreign reserves: acceleration of growth. Benigno and Fornaro (2012) build an endogenous growth model with knowledge spillovers through intermediate inputs and show how foreign reserves can increase imports of the inputs. They argue that by increasing foreign reserves, a real exchange rate is depreciated, and this leads high intermediate inputs. In addition, it also makes labor reallocate to tradable sectors, so the economy grows faster. This paper focuses more on imports while Dooley, Folkerts-Landau and Garber (2005) are interested on exports. They mention that high foreign reserves are a part of strategies to boost the nation's growth by keeping the foreign exchange rate undervalued. It seems to be a rational claim when we see countries like China and South Korea since their export-oriented growth strategy is widely known as their growth engine. As opposed to these papers, I build an endogenous growth model of a small open economy with knowledge flow through exporting to foreign markets.

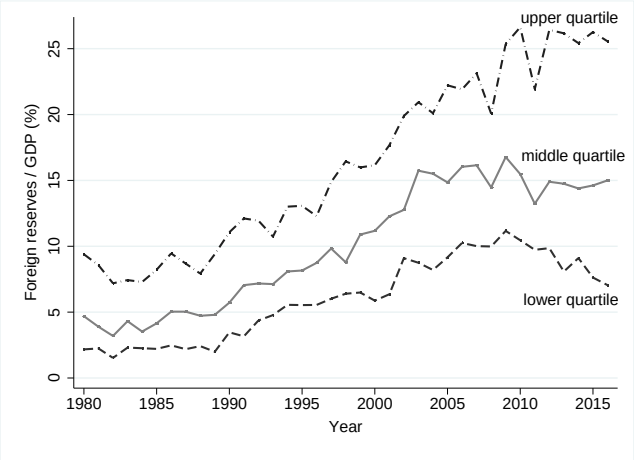
This paper is organized as follows: Section 2 describes empirical evidence on foreign reserves, exports and growth in developing economies; Section 3 presents a baseline model and defines the equilibria with and without a benevolent government to examine the role of the foreign reserve; Section 4 shows quantitative analysis to compare welfares; I then onclude in Section 5.

2 Data and empirical findings

This section describes definitions of main variables and documents a set of key findings. The data set contains annual data of 50 developing countries from 1980 to 2016. These countries were considered developing countries in 1995 according to the World Bank (1995) but are now classified as developed countries. [3] Since the paper analyzes the dynamic of growth, these countries, that used to be developing countries, are included in the data set. I

[3] World Bank classifies developing countries by low-income, middle-income, and the transition countries. South Korea, Slovenia, Czech Republic, Slovak Republic, Latvia, and Lithuania are now listed as high-income countries in World Bank (Fantom and Serajuddin (2016))

Figure 1: Quartile of foreign reserves to GDP



Notes: The dot-dash line presents the 3rd quartile (75 percentile) of the 50 sample countries. The gray solid line and the black dot lines are the median quartile (50 percentile) and the 1st quartile (25 percentile) of the sample respectively.

divide the sample countries into two types of countries: countries following a policy of export growth and countries without this policy. Detailed definitions will be explained in the next section.

Using the data, I document following facts.

1. Export-oriented countries have been accumulating foreign reserves more than others.

2. Export-oriented countries show more of a significantly positive correlation between growth rate and foreign reserves than others.

Before illustrating these facts, county growth policies are defined in the next section.

2.1 Export-oriented countries

Export-oriented countries are considered countries that have experienced high growth rate of exports-to-GDP ratio since 1980.[4] I assume that the countries who grow due to the export

[4]Exports-to-GDP ratio can be used to define the export-oriented economy, but some developing countries are small in terms of size of GDP. Thus, even though the absolute value of exports is very small, they could be included as a country with an export-growth policy. Also, China and Korea, which are well-known as the countries that follows a policy of export growth, are excluded with this definition. For this reason, I use the growth rate of exports-to-GDP ratio.

externalities increase exports more compared to their economic size, so the exports-to-GDP ratio shows an increasing trend. To capture this feature, I take the average of the exports-to-GDP ratio from 1980 to 1990 and also one from 2006 to 2016 and calculate the growth rate using the two values. The reason for taking the average for 10 years is to obtain the general level of the ratio in the beginning and end of the sample period.[5] Finally, the export-oriented countries are classified by the following equation.

$$g\left(\frac{Exports}{GDP}\right)_{1980-2016} > 0.51$$

Countries that grew more than 51% in the exports-to-GDP ratio compared to 1980s are classified as an export-oriented country. By this criterion 0.51 is at the 60th percentile in the sample. In other words, the top 40% of countries in terms of their growth rate are classified in this group. According to this definition, 19 countries are included in the export-growth policy countries.[6]

I also define non export-oriented countries as a comparative group to the export-growth policy countries. I assume that these countries tend not to increase their exports-to-GDP ratio because exports are not critical to growth. As a result, the criteria value is set to be 0.26, which is at the 40th percentile in the sample. In other words, the countries whose growth of exports-to-GPD ratio is less than 26% during the sample period are classified as non export-growth policy countries. The list of countries in two groups are shown in the Appendix(A).

The table(1) presents the mean values for main variables related to exports, GDP, and foreign reserves. The first three variables ($\frac{\text{Exports}}{\text{GDP}}$, $\frac{\text{Foreign Reserves}}{\text{GDP}}$, $g(\text{GDP})$) are the simple average over the sample period, from 1980 to 2016 and each column shows the mean of

[5]when a specific year is picked the general exports-to-GDP ratio might be misled if the country is hit by shocks. For example, the exports-to-GDP ratio of Argentina in 2001 picked up to 26% from 10%, but this occurred because the GDP dropped dramatically due to the economic crisis in 2001, not because of increase in exports.

[6]The median of the growth of the export-to-GDP ratio is 0.44.

Table 1: Mean values for the main variables

	All countries	Export-oriented	Non export-oriented
Exports/GDP	29.5%	28.7%	26.3%
Foreign reserves/GDP	11.1%	11.6%	9.0%
g(GDP)	2.1%	3%	1.2%
g(Exports/GDP)	65%	140%	2%
g(Foreign reserves/GDP)	5.1%	6.0%	4.5%

Notes: $g(\cdot)$ means the growth rate of a variable. Exports/GDP, Foreign reserves/GDP, and g(GDP) are the mean of each variable from 1980 to 2016. To obtain the mean value of the growth rate of exports-to-GDP ratio for each country I calculated two values for the exports-to-GDP ratio: one for the first 10 years and one for the last 10 year of the sample period. Then I obtained the growth rate using the two values. The number shown in each column is the mean value in each group. The mean values of the growth rate of foreign reserves-to-GDP ratio are also calculated in the same way.

countries in each group : all countries, export-growth policy, and non export-growth policy countries. The growth of export- and foreign reserves-to-GDP ratios are calculated according to the steps described above.

First of all, it is clear from this table(1) that mean values of the exports-to-GDP and the foreign reserves-to-GDP ratios do not show significant differences in mean of all countries. The average level of exports-to-GDP ratio is even higher for all countries. Thus, some countries that are not included in the export-oriented countries have higher exports. The export-oriented countries export more than non export-oriented countries. The foreign reserves-to-GDP ratio shows the same pattern. The foreign reserves-to-GDP ratio of all countries and export-oriented countries are quite similar while they have a bit higher level of foreign reserves than non export-oriented countries.

However, the average growth rates are different in the three groups. The growth rate of exports-to-GDP ratio is remarkably higher in the export-oriented countries. This is a natural result due to the definitions for the export-oriented countries. They grew more than 140 percent during the sample period while the non export-oriented countries only grew 2 percent. The mean for all countries is 65 percent. Also, countries following the export growth policy grew faster on average. This is consistent with the features of export-

oriented countries. If the export-oriented countries grow through exports, high exports imply high growth of the economy. Furthermore, those countries show the high growth of foreign reserves as well. This supports the main argument of this paper. As foreign reserves increase, countries export more, meaning higher growth of exports, and those countries grow faster. This is also related to the first empirical finding that I will document in the next section.

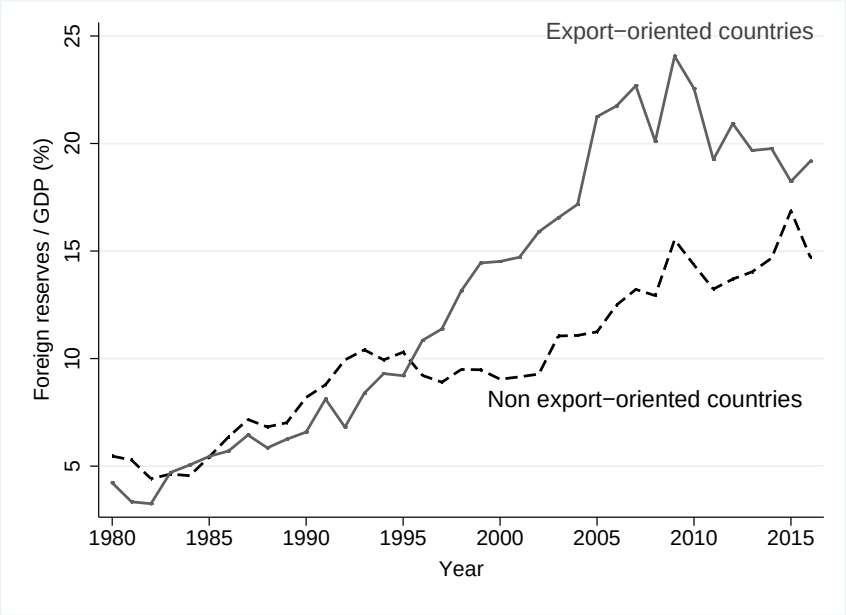
2.2 Foreign reserves and Exports

As shown in the table(1), the level of foreign reserves-to-GDP ratio in the export-oriented countries are higher than other countries. It is also worthwhile to see how foreign reserves change over time in two groups. If exports increase as foreign reserves are accumulated, then it can be concluded that the export-oriented countries accumulate foreign reserves faster than the non export-oriented countries. This fact is depicted in the figure(2). The gray solid line is the mean of foreign reserves-to-GDP ratio of export-oriented countries and the black dash line is that of non export-oriented countries.

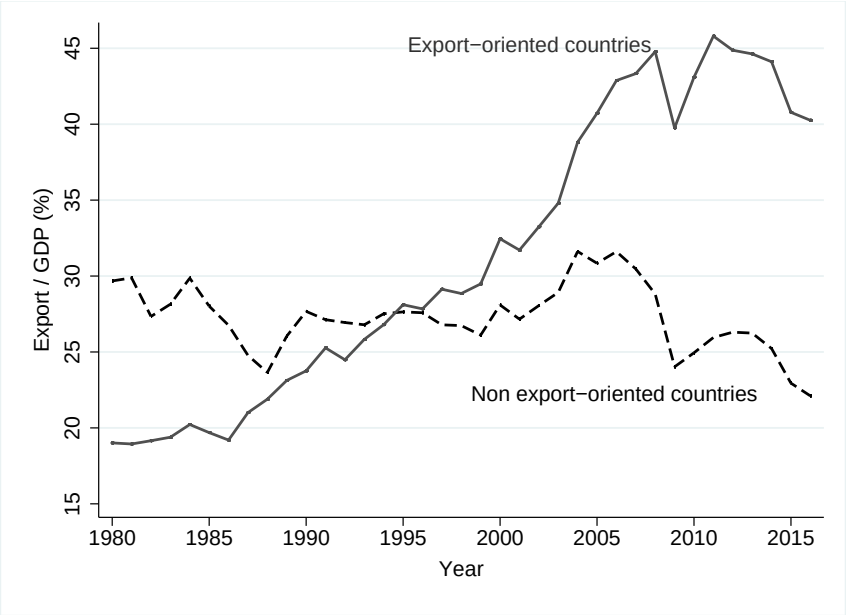
According to the first graph (a), The foreign reserves accounted for only around 5% in GDP in 1980 and they showed fairly similar pattern until 1995. During this period, the non export-oriented countries had even higher foreign reserves. Starting in 1995, the export-oriented countries began to overtake the group of other countries and kept increasing and reached over 20% right before 2007. This continuous growth of the foreign reserves suggests this importance as a policy in those countries. This is consistent with the idea that the countries that have export-led growth strategies can export more with higher foreign reserves. This argument is also supported by the graph (b). The increasing pattern of exports is quite similar to the foreign reserves in these countries. Both rose until the global crisis and decreased after 2010.

On the other hand, the average ratio in the non export-oriented countries is quite constant in 1990s. It started increasing in mid-2000s but failed to catch up to the foreign reserves-to-GDP in the countries with the export growth policy. Furthermore, there is no correlation

Figure 2: Share of GDP from 1980 to 2016



(a) Foreign reserves-to-GPD ratio



(b) Exports-to-GPD ratio

Notes: The gray solid line presents the mean of 19 countries following the export growth policy and the black dash line represents the mean of 18 countries without that policy.

between foreign reserves-to-GDP ratio and exports-to-GDP ratio in these countries. While the foreign reserves exhibit a rising pattern, the exports are constant and the movement

after 2005 even exhibits a decreasing pattern. From figure(2), it can be concluded that the export-oriented countries have higher level of foreign reserves, and the growth of the foreign reserves are faster.

2.3 Foreign reserves and Growth

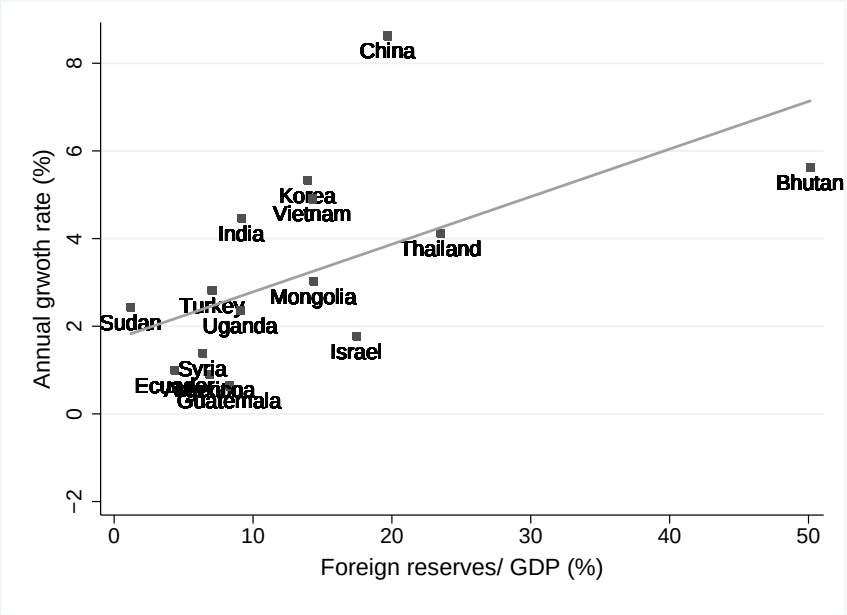
In addition to exports, economic growth also shows a positive relationship with foreign reserves in the export-oriented countries. Table(1) show that the export-oriented group has a higher growth rate of GDP per capita and also higher foreign reserves-to-GDP ratio compared to the other group of countries. Figure(3) provides a more detailed analysis of this fact using scatter plots. Plots (a) and (b) illustrate the relationship of annual growth rate and foreign reserves-to-GDP ratio in the export-oriented countries and non export-oriented countries respectively.

Countries having high foreign reserves experience faster growth if the countries are classified as an export-oriented country. The correlation coefficient is 0.56, indicating a significant positive correlation.[7] China displays the highest annual growth rate among those countries and is well-known as having export-led growth strategies. Furthermore, China has the most foreign reserves and it recorded 4 trillion dollars in 2014. This example supports the claim that high growth comes with high foreign reserves if the country follows the export growth policy. On the other hand, it is hard to say that there is a relationship between growth rate and foreign reserves-to-GDP ratio among the countries without the policy. Also, the correlation coefficient is 0.05, hardly different from 0.

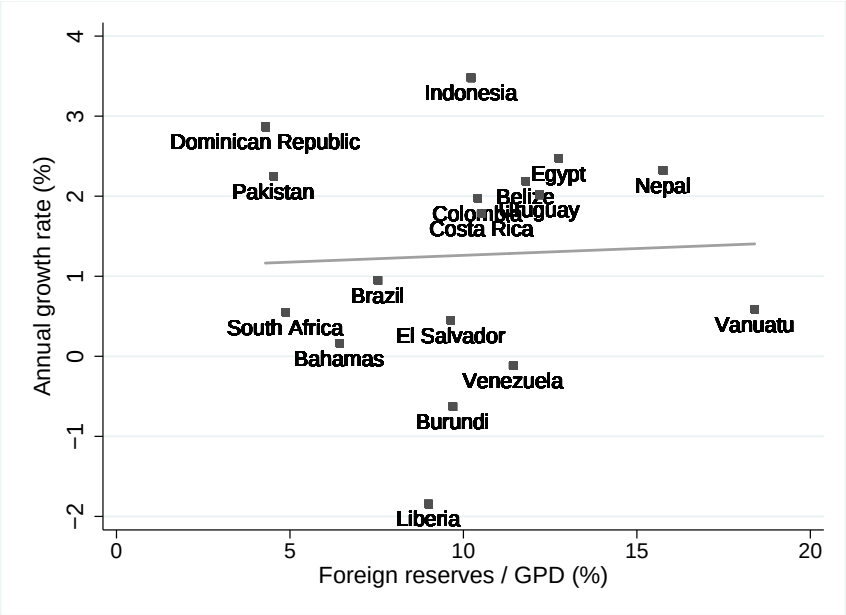
Additionally, even though the export-oriented countries tend to hoard higher foreign reserves compared to the other country group, the ranges are quite similar, spanning from 0 to 25 if Bhutan is excluded. However, the range of annual growth rate are noticeably different. No country in figure (a) records a negative growth rate and some countries such as China, Korea, Vietnam, India, and Thailand experienced high growth– more than 4% –

[7] Bhutan seems to be an extreme case but eliminating it will not change the positive correlation. Without Bhutan, the correlation coefficient is 0.62, which is higher than with it.

Figure 3: Growth rate and Foreign reserves-to-GDP ratio



(a) Countries with the export-growth policy



(b) Countries without the export-growth

Notes: Each plot shows the average value of annual growth rate and foreign reserves-to-GDP ratio of each country during the sample period.

from 1980. Their foreign reserves-to-GDP ratios lie between 15% to 25%. Meanwhile, the grow rates of non export-oriented countries within this range are less than 2%. Given the

fact that a 1% difference of annual growth rate can bring incredible differences over 37 years, the gap between the growth rates of the two groups is non-negligible. This indicates that foreign reserves are not correlated with growth rate in the non export-oriented countries, but the export-oriented countries, in fact, cannot ignore the effects of foreign reserves. They reap the benefits from the high foreign reserves.

I have currently supported two claims regarding foreign reserves, exports, and growth. The following section discusses a model and assumptions in detail and describes the modeling elements required to describe the empirical results of the previous section.

3 Model

In this section, I develop an endogenous growth model of an almost small open economy (SOE) in the presence of export externalities. There are domestic goods and foreign goods. The domestic goods are either consumed in the home country or exported to the foreign country. Similar to Benigno and Fornaro (2012), the knowledge spillovers of leading-edge technology of a foreign country take place through exports.[8] In this way, the home country accumluates the stock of knowledge that induces economic growth.

The relative price between foreign goods and domestic goods is defined as the terms of trade for the domestic goods are exported and foreign goods are imported. The export price is normalized to one so that terms of trade are effectively expressed in units of foreign goods.

This model is almost a small open economy in the sense that the interest rate of the foreign reserves and the foreign country's output are exogenously given. More precisely, the home country can affect the terms of trade, but not foreign income so this model is different from a small open economy. Also, the foreign country has deep pocket, so the interest rate of the foreign reserves is the risk free rate since it never defaults.

[8] Benigno and Fornaro (2012) explain the knowledge accumulation with imports. By importing foreign capital goods, firms in developing countries can benefit from the spillover effects of discovery from developed countries.

In this world, there are a home and a foreign country. Time is discrete and denoted by $t = 0, 1, \cdots, \infty$. In the home country there are three agents: the representative consumer, the representative firm, and the government. The consumer chooses how many domestic and foreign goods to consume. The firm produces domestic goods given the knowledge stock and sells them to the domestic consumer or exports them to the foreign country. Lastly, the government chooses how much to save in the foreign country. I call this external saving "foreign reserves" and this is a policy tool of the government in this economy. To see its role, I will propose two equilibria: the competitive equilibrium without the government and the equilibrium of the benevolent government problem.

3.1 Consumers

The consumers in this economy are infinitely lived and all identical. The representative consumer owns the domestic firm. She consumes a composite good of the domestic and the foreign goods. The preference is given by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{1}$$

where β is the discount factor and the utility function $u(\cdot)$ is the Constant-Relative-Risk-Aversion (CRRA), which has the functional form of $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ where σ is a risk aversion parameter. The composite good C_t is an Armington-type of CES aggregator with elasticity of substitution $1/(1-\eta)$ between domestic goods c^D and imported foreign goods c^F

$$c_t = c(c_t^D, c_t^F) = \left[\omega (c_t^D)^{-\eta} + (1-\omega)(c_t^F)^{-\eta}\right]^{-\frac{1}{\eta}} \tag{2}$$

where $0 < \omega < 1$ denote the share of expenditure in consumption that the consumer spends on the domestic tradable goods. The income sources of the consumer are wages w_t, profits from the firm π_t, and transfers from the government T_t. The total income is used to consume tradable and imported goods. Hence, the budget constraint is followed by:

$$c_t^D + p_t c_t^F = w_t L_t + \pi_t + p_t T_t \tag{3}$$

where p_t is terms of trade. I assume inelastic labor supply, so $L_t = 1$ for all t. The representative consumer receives transfers in terms of foreign goods since the government saves foreign reserves in the foreign country and they are denominated in foreign goods.

3.2 Firms

There is a large number of firms in this economy and all identical. The representative firm produces domestic goods that can be exported and is featured by knowledge spillovers. The firm produces using labor L_t, and the knowledge stock γ_t that governs the growth of the productivity of the economy. The production function is the following form:

$$Y_t = (\gamma_t L_t)^\alpha \tag{4}$$

The Y_t is the output of the domestic goods and α is labor share with $0 < \alpha < 1$.

The produced goods are sold in the domestic market and exported: $Y_t = q_t^d + x_t$ where q_t^d is domestic demand and x_t is export demand. Since the economy is almost small, the demand for domestic goods (exports) in the foreign country is given to the firm. The demand function depends on the terms of trade and outputs in the foreign country $Y^\star$ which is exogenously given.[9]

$$x_t = \left(\frac{1}{p_t}\right)^{-\phi} Y^\star \tag{5}$$

where $\phi > 0$. $-\phi$ is the elasticity of demand for domestic tradable goods.

[9] This demand function is standard in small open economy models and the function depends on export price p^x and world price $P^\star$ as follow: $x_t = \left(\frac{p_t^x}{P_t^\star}\right)^{-\phi} Y^\star$ In this paper, I use import price as the world price so $\frac{p_t^x}{P_t^\star} = \frac{1}{p_t}$ and the demand function is obtained

The firm's profit function is expressed as:

$$\pi_t = (\gamma_t L_t)^\alpha - w_t L_t \tag{6}$$

3.3 The Knowledge accumulation

The firm produces domestic goods given the available knowledge stock. I assume that the foreign country is more advanced, and the knowledge stock is non-rivalrous and non-excludable. These assumptions enable the knowledge spillovers from the foreign country to the home country. The key factor of the spillovers is exports. By exporting, the home country adopts new technologies and ideas made in the foreign country and the knowledge can be accumulated in this process. This feature is captured in the law of motion of the knowledge stock:

$$\gamma_{t+1} = \psi\gamma_t + x_t^{\zeta_x}\gamma_t^{\zeta_\gamma} \tag{7}$$

where $\psi \geq 0$ and $\zeta_x + \zeta_\gamma \leq 1$. γ_t is the knowledge stock in period t and ψ shows the persistency of the knowledge stock. ζ_x presents the share of knowledge that exports contribute so $\zeta_x = 0$ means no export externalities.

I assume that the externality function in the knowledge accumulation equation presents decreasing returns to scale, i.e, $\zeta_x + \zeta_\gamma \leq 1$. This assumption ensures that the optimal level of foreign reserves is finite.

3.4 The Government

The government trades foreign reserves and gives transfers to the consumer in this economy. The foreign reserves are one-period risk free bonds issued by the foreign country. The interest rate of the foreign reserves is R_f. The government receives returns from foreign reserves accumulated in the previous period A_t, chooses the level of foreign reserves A_{t+1}, and gives transfers T_t, to the consumers in a lump sum fashion. The government budget

constraint is given:

$$T_t = R_f A_t - A_{t+1} \tag{8}$$

4 Equilibrium

4.1 Competitive equilibrium without the government

In this section, consider the situation where there is no government in this economy. Thus, the foreign reserves and transfers are set to be zero every period $t \geq 0$. Substituting $T_t = 0$ into (3), the consumer's budget constraint becomes

$$c_t^D + p_t c_t^F = w_t L_t + \pi_t \tag{9}$$

The consumer chooses a consumption bundle $\{c_t^D, c_t^F\}_{t\geq 0}$ to maximize the expected value of utility (1) subject to (9), taking prices, such as terms of trade and wage, and profits $\{p_t, w_t, \pi_t\}_{t\geq 0}$ as given. The consumer's optimality condition is:

$$p_t = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} \tag{10}$$

$u_i(c_t^D, c_t^F)$ means a partial derivative of the utility function with respect to the ith argument at time t. The equation (10) indicates that the terms of trade (relative price) equals the marginal rate of substitution of the foreign and the domestic goods.

In the firm's problem, the firm chooses $\{Y_t, L_t, q_t^d, x_t, \pi t\}_{t\geq 0}$ given terms of trade $\{p_t\}_{t\geq 0}$ and knowledge stock $\{\gamma_t\}_{t\geq 0}$. As mentioned in the previous section, there is a large number of firms so each firm is too small to internalize the effects of externalities. Therefore, the firm takes the knowledge stock as given every period.

Furthermore, the firm's problem is not a maximization problem. Given the knowledge stock and inelastic labor supply, the output of the domestic goods is determined and the level of exports is determined by the foreign demand function. Consequently, the amount of

domestically consumed goods and the profits are automatically determined. The first-order condition of the firm is only for the labor demanded:

$$w_t = \alpha\gamma_t(\gamma_t L_t)^{(\alpha-1)} \tag{11}$$

The optimality condition (11) equates wage and marginal product of labor.

Market clearing for domestic goods requires that the amount produced is equal to the amount consumed in the home and the foreign country. Since consumers are identical, the market clearing condition is:

$$c_t^D + x_t = Y_t \tag{12}$$

Notice that the domestic goods that are consumed in the home are equal to those produced for the domestic market, so the market clearing condition (12) already reflected $c_t^T = q_t^d$. Combining (12) with the consumer's budget constraint (9) and the firm's profits function (6), the trade balance is obtained as :

$$x_t - p_t c_t^F = 0 \tag{13}$$

Using the optimality conditions and the market clearing conditions, the competitive equilibrium can be defined

Definition 1. ***Competitive Equilibrium***
The competitive equilibrium for the almost small open economy is defined by a set of the representative consumer's allocations $\{c_t^D, c_t^F\}_{t\geq 0}$, the representative firm's allocations $\{Y_t, L_t, q_t^d, x_t, \pi_t\}_{t\geq 0}$, the terms of trade and the wages $\{p_t, w_t\}_{t\geq 0}$, and the stock of knowledge $\{\gamma_{t+1}\}_{t\geq 0}$, given initial knowledge stock and foreign output $\{\gamma_0, Y^\star\}$ such that the following conditions are satisfied:

1. *Consumer: Given $\{p_t, w_t, \pi_t\}_{t\geq 0}$, the consumer's allocations $\{c_t^D, c_t^F\}_{t\geq 0}$ satisfy the optimality condition (10).*

2. *Firm: Given $\{p_t, w_t, \gamma_t, Y^\star, z\}_{t\geq 0}$, the firm chooses $\{Y_t, L_t, q_t^d, x_t, \pi_t\}_{t\geq 0}$ satisfying (4), (5), (6), and (11).*

3. *The market for domestic goods clears as (12) and trade balance satisfies (13).*
4. *The knowledge stock evolves according to (7).*

4.2 The Benevolent Government

In the previous section, the competitive equilibrium without the government was described. Now, consider the benevolent government who maximizes the consumer's utility. The government makes a foreign reserves decision which is not allowed to the private agents and pays (or collects) transfers (taxes) to the consumer in a lump-sum fashion. Then, the consumers choose their consumption bundle of the domestic and the foreign goods in a competitive way. Thus, the consumer's budget constraint in this benevolent government problem is the equation (3) including transfers T_t. Also, the trade balance should be different from that with no government case. Combining the government budget constraint (8) with the consumer's budget constraint (3), the trade balance is expressed as:

$$x_t - p_t c_t^F = p_t \left(A_{t+1} - R_f A_t \right) \tag{14}$$

One important feature of the government is that it can internalize the impact of growth through the knowledge accumulation. This aspect is a significant difference from the private agents in this model. The government realizes that choosing a higher level of foreign reserves induces higher terms of trade and it is advantageous for high exports. Since the knowledge stock can be accumulated through high exports, the home country can experience faster growth by hoarding foreign reserves.

Now consider the optimization problem of the benevolent government in a recursive form. The state variables are the current foreign reserves holding A and the knowledge stocks γ. The crucial state variable is the later one. Since the representative agents do not internalize the knowledge stocks, the competitive equilibrium is solved without taking account of the dynamic process of knowledge accumulation. However, the benevolent government realizes it so takes the knowledge stocks as a state variable. Combining the budget constraint (3)

and (14), the resource constraint for the tradable good can be expressed as

$$c_t^D + x_t = (\gamma_t L_t)^\alpha \tag{15}$$

The benevolent government optimization problem consists of maximizing the value of consumer's life time utility (1) subject to the resource constraint for the tradable goods (15), the trade balance (14), the consumer's optimality condition (10), the demand function of exports in the foreign country (5), and the law of motion of knowledge stock (7). The government's recursive optimization problem can be expressed as following:

$$V(A,\gamma) = \underset{c^D, c^F, c, x, A'}{Max} \{u(c(c^D, c^F)) + \beta V(A', \gamma')\} \tag{16}$$

$$\begin{aligned}
\text{s.t} \quad & c^D + x = z\,(\gamma L)^\alpha \\
& x - pc^F = p\,(A' - R_f A) \\
& x = \left(\frac{1}{p}\right)^{-\phi} Y^\star \\
& \gamma' = \psi\gamma + (x)^{\zeta_x}(\gamma)^{\zeta_\gamma} \\
& p = \frac{1-\omega}{\omega}\left(\frac{c^D}{c^F}\right)^{1+\eta}
\end{aligned} \tag{17}$$

Where $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ and $c(c^D, c^F) = \left[\omega(c^D)^{-\eta} + (1-\omega)(c^F)^{-\eta}\right]^{-\frac{1}{\eta}}$

where the notation for variable with the prime superscript means variable in the next period. As shown in the problem the government knows the optimal actions of the private agents, and chooses an optimal allocation of consumption $\hat{c}^D(A,\gamma), \hat{c}^F(A,\gamma)$, exports $\hat{x}(A,\gamma)$, and foreign reserves $\hat{A}'(A,\gamma)$ to maximize the consumer's utility.

Definition 2. ***Recursive Equilibrium***
The recursive equilibrium for the benevolent government is defined by a pricing function $\hat{p}(A,\gamma)$, and a decision rule $\hat{A}'(A,\gamma)$ for the benevolent government with associated value function $V(A,\gamma)$, consumption and exports rules $\{\hat{c}^D(A,\gamma), \hat{c}^F(A,\gamma)\}$, $\hat{x}(A,\gamma)$, and a knowledge stock rule $\hat{gamma}(A,\gamma)$ such that the following conditions hold:

1. *Given $\hat{p}(A,\gamma)$, the decision rule $\hat{A}'(A,\gamma)$ solves the benevolent government recursive maximization problem (16) and the exports policy $\hat{x}(A,\gamma)$ satisfies (5).*

2. *Given $\hat{p}(A,\gamma)$, the tradable consumption plan $\hat{c}^D(A,\gamma)$ satisfies the resource constraint (15) of the economy and the imported goods consumption plan $\hat{c}^F(A,\gamma)$ satisfies the trade balance (14).*

3. *The knowledge stock $\hat{\gamma}(A,\gamma)$ evolves according to (7).*

4.3 Externalities and Efficiency

In the previous sections, I described the competitive equilibrium under no government and the recursive equilibrium of the benevolent government problem. Since the private agents cannot internalize the export externalities, the competitive equilibrium without the government cannot achieve the first best. However, the government can take into account the externalities, thus it will use its policy tool to make the private agents achieve the second best. I illustrate how the externalities are considered in the government problem.

Consider the first order condition (30 for the benevolent government. Appendix (B) provides full parts of the characterization of the benevolent government allocation. This equation shows the crucial difference between the equilibrium solved by the benevolent government and that of the competitive equilibrium. The sequential notation is used to compare two equilibria.

$$p_t = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)\left(1 - \Psi_t\left(p_t^{-\phi(1-\zeta_x)}\right)\right)} \tag{18}$$

where $\Psi_t = \alpha\zeta_x\left(\gamma_t\right)^{\zeta_\gamma}\left(\frac{u_1(c_{t+1}^D, c_{t+1}^F)}{u_1(c_t^D, c_t^F)}\left(\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right) > 0$.

When $\zeta_x = 0$, the equation (18) equals the corresponding equation for the competitive equilibrium (10). The (10) equates the terms of trade to the marginal rate of substitution; however, the government considers the additional effects. The denominator on the right-hand side is divided into two terms: the marginal utility of consuming one unit of domestic good ,$u_1(c_t^D, c_t^F)$, and the marginal benefits of exporting one unit of tradable good, $\Psi_t\left(p_t^{-\phi(1-\zeta_x)}\right)$. The first term presents the increase in the marginal utility of the domestic goods today while

the second term includes the marginal utility tomorrow. This term represents the benefits of increased knowledge stock implied by an additional export of the domestic goods. One more unit of export today induces higher knowledge tomorrow, and thus more output will be produced with a higher productivity tomorrow. As a result, the consumers will be able to consume more domestic goods tomorrow. In other words, this term refers to growth effects from the export externalities that are not internalized by the private agents. Furthermore, the benefits are realized in the next period. Then, due to the market clearing condition the government has to choose either to consume or to export. If the consumer decides to consume the domestic good today, it has to give up the benefits of growth so the marginal utility of domestic good is $u_1(c_t^D, c_t^F)\left(1 - \Psi_t\left(p_t^{-\phi(1-\zeta_x)}\right)\right)$. As the export externalities, ζ_x increase the benefits of exporting becomes larger so the marginal utility of domestic goods becomes smaller. Equivalently it is a decision of the increase in utility between today and tomorrow. Since the government internalizes the growth effects from the exports, it has an incentive to use the foreign reserves as a policy tool to stimulate exports.

To see how the government uses the foreign reserves, compare the terms of trade in the two problems. Define p_t^c and p_t^g the terms of trade under the competitive equilibrium and benevolent government problem respectively.

$$p_t^c = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} \quad , \qquad p_t^g - \Psi_t (p_t^g)^{1-\phi(1-\zeta_x)} = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} \tag{19}$$

Since $\Psi_t > 0$ and $p_t^g > 0$, the terms of trade under the government problem are higher than that under the competitive equilibrium, $p_t^c < p_t^g$ if both has the same consumption bundle. The government takes into account the growth effects through exporting, so the opportunity cost of the domestic goods becomes higher under the government problem. Hence, the cost of consumption of the foreign goods is relatively expensive. As a result, the government wants the market to achieve p_t^g. The lower terms of trade in the competitive environment mean that too many foreign goods are consumed. Thus, by accumulating foreign reserves in terms of foreign goods, the government makes the consumer consume fewer foreign goods,

and thus the higher terms of trade can be obtained. This is how the government manages foreign reserves.

5 Quantitative Analysis

Here I study the quantitative implication of the model using a baseline calibration based on data from Korea, well known as an export-oriented country hoarding sizable foreign reserves. I investigate the mechanism of the model and provide a welfare analysis over the sample period.

5.1 Calibration

The parameters in the model are calibrated with quarterly frequency. Table(2) shows the calibrated parameter values. The weight on domestic goods in utility function ω is set by the ratio of consumption on tradable goods to total consumption on tradable goods averaged over the sample period. Tradable goods are defined as non-service goods and the domestic tradable consumption is obtained by subtracting import of consumption goods from total consumption of tradable goods.

The risk aversion parameter σ is set to 2 and the quarterly world risk-free interest rate R_f is set to 1%, which are standard values in international macro studies. The curvature of Armington aggregate of domestic tradable goods and imported goods η is set to be 0.11, following the calibration of Heathcote and Perri (2002). This implies the elasticity of substitution between domestic goods and foreign goods is 0.9. According to Kabaca (2014), Korea's labor share in output α was 0.70 during the period from 1980 to 2008. The value of ϕ is obtained from Imbs and Mejean (2010). The elasticity of demand for exports varies by countries.

Finally, the constant term in the knowledge process ψ is calibrated. The target moment

Table 2: Parameters

Parameter	Description	Value	Source/Data
β	Discount factor	0.96	4% interest rate
σ	Risk aversion	2.00	Standard value
ω	Weight on domestic tradable goods	0.64	$\frac{1}{T}\sum$ (Non-import cons./ Tradable cons.)
η	Amington curvature	0.11	Heathcote and Perri (2002)
α	Labor share in output	0.70	Kabaca (2014)
ϕ	Elasticity of export demand	1.50	Imbs and Mejean (2010)
ψ	Constant in knowledge process	0.34	5% Annual GDP growth
ζ_x	Export externality parameter	0.70	
ζ_γ	knowledge contribution parameter	0.20	
R_f	Gross risk-free interest rate	1.01	standard value

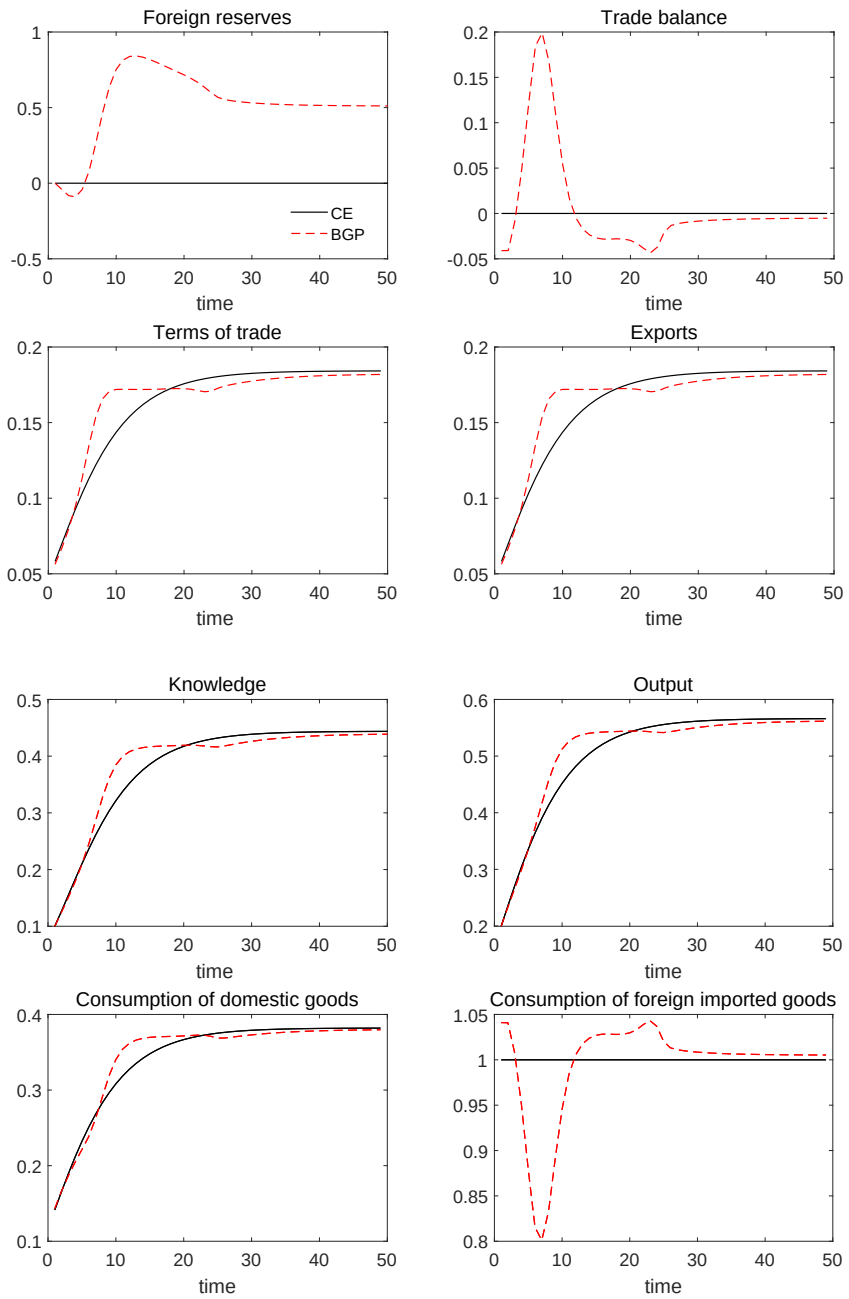
is the annual growth rate of GDP during the sample period, which is 5% and $\psi = 0.34$. For now, to check the mechanism of the model, I assume high export externalities, so set $\zeta_x = 0.7$ and $\zeta_x + \zeta_\gamma = 0.9$, that ensures that the optimal foreign reserves is finite.

5.2 Results

Based on this calibration, I analyze the solutions of two equilibria: the competitive equilibrium and the equilibrium of the benevolent government. I simulate the models to analyze the growth of knowledge, output, and consumption in the long run and figure (4) shows the two equilibria. The black solid line represents the competitive equilibrium without the government and the red dash line displays the equilibrium of the benevolent government's problem. The initial foreign reserves and the initial knowledge stock is 0 and 0.1 respectively and I simulate for 50 periods.

I first show the simulation results from the competitive equilibrium without the government. The private agents are not allowed to save in the foreign country and there is no government in this equilibrium, the foreign reserves are zero all the time. This also implies zero trade balance. The economy exports as much as imports times terms of trade. Given the

Figure 4: Simulation results



Notes: black solid lines are for the competitive equilibrium without the government intervention and red dash lines are for the benevolent government.

initial knowledge stock, the economy accumulates knowledge by exporting domestic goods. Hence, knowledge stock grows over time, and thus the output also grows since the knowledge

stock is an input factor of the production. As output increases, the consumption of domestic and foreign goods increase. The terms of trade is less than 1. This means the domestic goods are more expensive than foreign goods. For this reason, the consumption of domestic goods is less than the consumption of foreign goods. Eventually, this economy converges.

On the other hand, internalizing the effects of knowledge externalities, the benevolent government accumulates foreign reserves over time. During the first few periods, the government saves in the foreign country very quickly, so the foreign reserves sharply increase. Meanwhile, the government policy induces the higher terms of trade and this enables the economy to stimulate exports. Thus, the knowledge stock increases as the exports rise. When the knowledge converges to the steady state value, the government does not need to boost exports any more. This is a consequence of decreasing returns to scale of the externality function in the law of motion of knowledge. Larger exports give lower marginal benefits of exporting, and thus the knowledge does not significantly grow at the certain level of exports. Thus, after the knowledge converges, the government attempt to keep exports as much as maintaining the steady state knowledge stock. Therefore, the foreign reserves show overshooting during the first few periods to boost exports but decrease to the point where can keep the steady state value of the knowledge stock.

The accumulation of the foreign reserves results in the lower consumption of both goods compared to the competitive equilibrium. The consumption on the foreign goods is lower because the government collects the lump-sum taxes in terms of the foreign goods from the consumer to finance the foreign reserves. In Furthermore, the consumption on the domestic goods is also lower because more domestic goods are exported due to the accumulation of foreign reserves. However, higher foreign reserves generate higher knowledge stock and output in the future, so the economy grow faster. This explains why from period 10 the consumption of both goods becomes higher than that in the competitive equilibrium. Since the economy grows faster consumers consume more with higher output. This shows the government takes into account the growth effects from the export externalities

To describe the impact on welfare of the government policy, I compute the welfare gains from accumulating foreign reserves. Define CE as the competitive equilibrium with no government and BG as the equilibrium solved by the benevolent government. The welfare gain is measured by the effect of considering the externalities that make the CE equal to the BG in proportion to the consumption in the CE. This calculation explicitly takes into account the cost of reducing consumption when switching to an economy without foreign reserves. Welfare gains from accumulating foreign reserves, θ defined as

$$\sum_{t=0}^{\infty} \beta^t u((1+\theta^{CE})c_t^{CE}) = \sum_{t=0}^{\infty} \beta^t u(c_t^{BG}) \tag{20}$$

With the calibrated values, the welfare gains from accumulating foreign reserves are 0.22% of permanent consumption when the foreign reserves to GDP ratio is 0.9. This ratio is very high compared to the foreign reserves to GDP ratio in Korea, 0.26. This is because I set the large number for the export externalities.

5.3 Export externalities and foreign reserves decisions

In the previous section, I show the case where the government accumulates foreign reserves with the calibrated parameters. However, it is not always optimal for the government to accumulate foreign reserves. The optimal level of the foreign reserves is determined by comparing the growth effects through exports and the benefits of the consumption smoothing and the tariff policy. The magnitude of the two effects depends on the size of the export externalities. Suppose the export externalities are large, then exporting the domestic goods significantly increases the level of knowledge stock in the next period. With the high knowledge stock, the economy can produce more domestic goods tomorrow. Thus, the economy experiences the faster growth by sacrificing today's consumption of the domestic goods. In this regard, the government has an incentive to hold positive foreign reserves to induce the

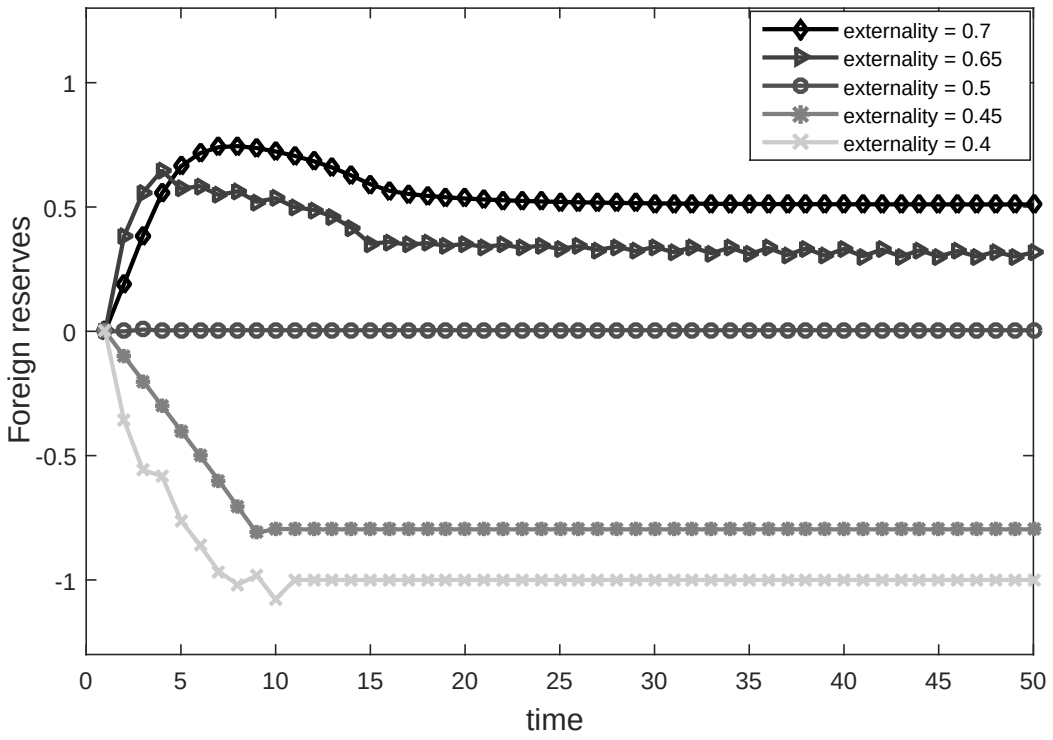
high terms of trade, which can lead to the high exports for the fast growth.

On the other hand, when export externalities are small, the benefits from the consumption smoothing motivation and improvement of terms of trade is stronger than the growth effects. Therefore, the government holds negative foreign reserves, which is borrowing from abroad at the risk-free interest rate. By borrowing in terms of foreign goods the terms of trade become lower, meaning that imported foreign goods become cheaper so the consumers consumer more foreign goods today. Also, lower terms of trade imply that the government decides not to stimulate exports and consume more domestic goods today as well as imported goods. This is related to the consumption smoothing motivation because the agents know that the economy will have higher income tomorrow, so it borrows foreign goods today. In addition, similar to the optimal tariff policy, the government induces better terms of trade to increase the consumer's welfare by borrowing foreign goods.

This mechanism is shown in figure (5). This graph illustrates the optimal foreign reserves level depending on the size of the export externalities. I vary the externality parameter ζ_x and see the government decisions on foreign reserves. The baseline model has externality parameter 0.7 and the foreign reserves reach to 0.74 in period 10 and decrease to 0.51 which is the steady state level. As the size of foreign reserves become smaller, the optimal level of foreign reserves also decreases. When the externality parameter is 0.5, the government is almost indifferent between intervention and no intervention. Once the externality parameter is less than 0.5, consumption today gives higher welfare than exporting, so the government borrow to improve terms of trade.

This exercise provides the different foreign reserves policy implications in developing countries. The countries having high export externalities would hold high foreign reserves to enhance exports through the deterioration of the terms of trade. On the other hand, the countries having low export externalities would receive capital inflow from the foreign countries to gain welfare improvement through consumption smoothing and better terms of trade.

Figure 5: Export externalities and Foreign reserves decisions



Notes: Externality is the export externality parameter ζ_x. Each line represents the optimal foreign reserves obtained by solving the benevolent government problem with different externality parameter.

5.4 Competitive equilibrium with private savings

I showed the optimal actions of the private agents are less efficient than the benevolent government. To achieve higher welfare the government uses foreign reserves. Then it is natural to consider that the existing government's external saving technology might be the source of efficiency because the private agents is not allowed to save. Hence, in this section, I discuss the welfare gains just from internalizing the externalities by comparing the competitive equilibrium with private savings and no government to the equilibrium of the benevolent government.

All environments are the same except the consumer's problem. The consumer now can save foreign goods in the foreign country by buying foreign bonds with the risk-free interest

rate. This will change two equations compared to competitive equilibrium without savings: the consumer's budget constraint and the trade balance. The budget constraint of the consumer includes savings s. These are the external savings, so it must be saved in terms of foreign goods. Since the government does not exist, transfers are dropped from the budget constraint. The following equation is the consumer's budget constraint with savings.

$$c_t^D + p_t(c_t^F + s_{t+1}) = w_t L_t + \pi_t + p_t R_f s_t \tag{21}$$

Also, substituting (21) and (6) into (12), the trade balance becomes

$$x_t - p_t c_t^F = p_t \left(s_{t+1} - R_f s_t\right) \tag{22}$$

which is similar to the benevolent government trade balance. The definition and the characterization of the competitive equilibrium for this economy with private saving is defined in appendix (C).

In this setup, the private agents will sell the foreign bonds i.e., borrowing rather than buy them i.e., lend. The consumers know that they will have a higher output tomorrow due to the accumulation of knowledge. Hence, the consumers will borrow to smooth their consumption. In fact, this action badly affects the growth of the economy. The borrowing in the private sector deteriorates the terms of trade, so the economy exports less and imports more. This hinders knowledge accumulation, and thus the economy grows slowly. This situation could be opposite of the action of the benevolent government. If the future growth benefits from the export externalities outweigh the utility from consuming the foreign goods today, the government would save. On the other hand, not internalizing the externalities, the private agents would borrow. Therefore, opening capital account is not favorable to the government and the accumulation of the foreign reserves generates higher welfare gain than the competitive equilibrium with no private saving.

6 Conclusion

In this paper, I propose an endogenous growth model of a small open economy with export externalities that provides a plausible explanation for different foreign reserves policy in various countries. I argue that some countries with high export externalities can accelerate the economic growth through technological spillovers from advanced countries by accumulating foreign reserves.

The key factor in the model is knowledge spillovers through exports, and it can be internalized by the government, but not by private agents. The consumers and the firms do not consider the growth effects from the exports so the exports would be less than the socially optimal level if there is no government policy. However, the government takes into account the export externalities that induce faster growth of the economy, and thus it uses the foreign reserves as a policy tool to manipulate the terms of trade to stimulate exports. By increasing the foreign reserves, the terms of trade are deteriorated, so the price of the domestic goods become cheaper. This leads to higher foreign demand for exports so export more and the home country can grow faster.

However, this mechanism works only if the export externalities are large enough. When the export externalities are positive but small, the benevolent government would hold negative level of foreign reserves, implying borrowing from abroad at the risk-free interest rate. This is because the benefits of the consumption smoothing motivation and better terms of trade by borrowing foreign goods outweigh the growth effects through exports. Therefore, the government has an incentive to improve the terms of trade to increases its welfare.

I calibrated the model with data from South Korea and showed the optimal choice for the foreign reserves. I found that the government accumulates foreign reserves faster until the economy reaches to the steady state level. During this period, the terms of trade are high, and shows faster growth of the economy through higher exports. Once it converges to the steady state, it is not necessary to enhance the exports anymore, so the optimal foreign reserves are determined at the level that maintains the steady state level of the economy.

The welfare gain from accumulating foreign reserves is 0.22% of permanent consumption in the economy with no government policy.

Also, I vary the externality parameters to examine the different optimal level of foreign reserves. The results show that as the export externalities become smaller, the growth effects through exports become smaller too, and thus the optimal level of foreign reserves decreases. Therefore, even though the export externalities are positive, the government chooses to hold negative foreign reserves. When I allow private saving, the welfare gain would be larger if the export externalities large enough, since the private agents who do not internalize the externalities would borrow by expecting higher income in the future.

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Appendices

A Data

All data is annual data from 1980 to 2016. The data availability varies by countries so countries with missing data on their exports, real GDP are dropped from the sample. Consequently, the final sample consists of 50 countries. All data is obtained from the World Development Indicators (WDI).

- Exports : The dollar value of exporting goods and services.
- Growth rate of GDP: Growth rate is calculated with Growth rate of real GDP per capita. The real GDP is expressed in local currency.
- Foreign reserves: Gross foreign exchange reserves minus gold.

A.1 Export-orientation

The following table (3) and table (4) list export-oriented countries and non export-oriented countries in the sample. It presents the growth of exports to GDP ratio.

Table 3: countries with a export growth policy

Country	$g\left(\frac{Exports}{GDP}\right)$	$\left(\frac{Exports}{GDP}\right)_{1980}$	$\left(\frac{Exports}{GDP}\right)_{2016}$
Argentina	0.99	0.09	0.18
Bhutan	1.34	0.18	0.42
Bolivia	0.73	0.23	0.40
Burkina	1.11	0.10	0.20
China	1.71	0.10	0.27
Ecuador	0.66	0.17	0.28
Guatemala	0.52	0.16	0.24
Hong Kong	0.95	1.02	2.00
India	2.74	0.06	0.22
Israel	1.31	0.11	0.25
Korea	0.65	0.29	0.48
Mexico	0.87	0.16	0.31
Mongolia	0.85	0.27	0.50
Sudan	0.98	0.07	0.15
Syria	1.48	0.16	0.39
Thailand	1.65	0.26	0.69
Turkey	0.68	0.13	0.22
Uganda	0.62	0.12	0.19
Vietnam	6.70	0.10	0.78

Table 4: countries without a export growth policy

Country	$g\left(\frac{Exports}{GDP}\right)$	$\left(\frac{Exports}{GDP}\right)_{1980}$	$\left(\frac{Exports}{GDP}\right)_{2016}$
Bahamas	-0.42	0.64	0.37
Belize	0.06	0.55	0.59
Brazil	0.20	0.10	0.12
Burundi	-0.25	0.10	0.08
Colombia	0.15	0.14	0.17
Costa Rica	0.11	0.31	0.35
Dominican Republic	0.02	0.24	0.25
Egypt	-0.05	0.22	0.21
El Salvador	0.22	0.22	0.27
Indonesia	-0.01	0.26	0.25
Liberia	-0.43	0.62	0.35
Nepal	-0.02	0.11	0.11
Pakistan	0.03	0.12	0.12
Senegal	-0.02	0.27	0.27
South Africa	0.09	0.28	0.31
Uruguay	0.20	0.21	0.26
Vanuatu	0.04	0.42	0.43
Venezula	0.10	0.24	0.27

B Benevolent government allocation

In this appendix, the benevolent government allocations are characterized. The benevolent government chooses $\{c_t^D, c_t^F, c_t, x_t, A_{t+1}\}_{t\geq 0}$ to solve the benevolent government problem (16) The first order conditions of the government's problem are

$$\beta^t u_1(c_t^D, c_t^F) = \lambda_t \tag{23}$$

$$\beta^t u_2(c_t^D, c_t^F) = \mu_t \tag{24}$$

$$\beta R \left(\frac{p_{t+1}}{p_t}\right) \mu_{t+1} = \mu_t \tag{25}$$

$$\frac{\mu_t}{\lambda_t} = p_t - p_t \left[\alpha\zeta(\gamma_t)^{\zeta_\gamma}(x_t)^{1-\zeta_x}\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right] \tag{26}$$

λ_t and μ_t are the Lagrange multiplier associated with (12) and (14) respectively. Substituting (23) and (24) into (26) and using the functional forms for the CRRA utility and the CES aggregate consumption, I can obtain

$$\frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} = p_t - p_t \left[\alpha\zeta(\gamma_t)^{\zeta_\gamma}(x_t)^{1-\zeta_x}\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right] \tag{27}$$

$$= p_t - p_t \left[\alpha\zeta(\gamma_t)^{\zeta_\gamma}(x_t)^{1-\zeta_x}\left(\frac{u_1(c_{t+1}^D, c_{t+1}^F)}{u_1(c_t^D, c_t^F)}\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right] \tag{28}$$

$$= p_t - p_t \left[(p_t)^{-\phi(1-\zeta_x)}\alpha\zeta(\gamma_t)^{\zeta_\gamma}\left(\frac{u_1(c_{t+1}^D, c_{t+1}^F)}{u_1(c_t^D, c_t^F)}\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right] \tag{29}$$

$$= p_t \left(1 - p_t^{-\phi(1-\zeta_x)}\Psi_t\right) \tag{30}$$

where $\Psi_t = \alpha\zeta(\gamma_t)^{\zeta_\gamma}\left(\frac{u_1(c_{t+1}^D, c_{t+1}^F)}{u_1(c_t^D, c_t^F)}\frac{Y_{t+1}}{\gamma_{t+1}}\right)$ The (30) is equivalent to the equation (18) and the management of foreign reserves are implied by this expression.

C Competitive Equilibrium allocation for the economy with private savings

This appendix provides the characterization of the competitive equilibrium for the economy with private savings. The consumer chooses $\{c_t^D, c_t^F, c_t, s_{t+1}\}_{t\geq 0}$ and the first order conditions are

$$p_t = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} \tag{31}$$

$$\beta R \left(\frac{p_{t+1}}{p_t}\right) \lambda_{t+1}^s = \lambda_t^s \tag{32}$$

$$\tag{33}$$

where λ_t^s is the Lagrange multiplier associated with (21). The equation (31) equates the terms of trade and marginal rate of substitution and the equation (32) is the Euler equation.

The definition of competitive equilibrium for this economy is shown below.

Definition 3. ***Competitive Equilibrium***
The competitive equilibrium for the small open economy with private saving is defined by a set of the representative consumer's allocations $\{c_t^D, c_t^F, s_{t+1}\}_{t\geq 0}$, the representative firm's allocations $\{Y_t, L_t, q_t^d, x_t\}_{t\geq 0}$, the terms of trade and the wages $\{p_t, w_t\}$, and the stock of knowledge $\{\gamma_{t+1}\}_{t\geq 0}$, given initial savings, initial knowledge stock, foreign output, and productivity, $\{s_0\gamma_0, Y^\star, z\}$ such that the following conditions are satisfied:

1. *Consumer: Given $\{p_t, w_t, \pi_t\}_{t\geq 0}$, the consumer's allocations $\{c_t^D, c_t^F, s_{t+1}\}_{t\geq 0}$ stratifies optimality condition (31).*
2. *Firm: Given $\{p_t, \gamma_t, Y^\star, z\}_{t\geq 0}$, the firm chooses $\{Y_t, L_t, q_t^d, x_t, \pi_t\}_{t\geq 0}$ satisfying (4), (5), (6), and (11).*
3. *The market for tradable goods clears as (12) and trade balance satisfies (22).*
4. *The knowledge stock evolves according to (7).*